

3 (Sem-2) PHY M 1

2 0 1 9

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods—II)

(Marks : 35)

1. Answer the following questions : 1×3=3

(a) State the condition for which any vector \vec{A} is conservative.

(b) Define unit vector in curvilinear coordinate system.

(c) Evaluate $\nabla \cdot \vec{0}$.

2. If $\vec{F} = \cos y \hat{i} - x \sin y \hat{j} - \cos z \hat{k}$, show that the field is conservative.

2

(2)

3. Answer any two of the following questions :

5×2=10

- (a) (i) Find the work done in moving an object in the field

$$\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$

from (1, -2, 1) to (3, 1, 4). 3

- (ii) Show that the area bounded by a simple closed curve C is given by

$$\frac{1}{2} \oint_C (x dy - y dx) \quad 2$$

- (b) (i) Express $\vec{\nabla} \times \vec{A}$ in spherical coordinates. 2

- (ii) Write down the transformation equations of orthogonal cylindrical and spherical coordinate systems from Cartesian coordinate and provide the limit of different parameters. 3

- (c) (i) Evaluate

$$\int_0^{\infty} \frac{x^a}{a^x} dx$$

using gamma function. 2

- (ii) Show that

$$\delta(ax) = \frac{1}{|a|} \delta(x)$$

for any non-zero value of a . 3

A9/761

(Continued)

(3)

4. Answer any two of the following questions :

10×2=20

- (a) (i) If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, evaluate

$$\iiint_V \vec{F} dV$$

where V is the region bounded by the surfaces $x=0, y=0, x=2, y=4, z=x^2, z=2$. 5

- (ii) A vector field \vec{F} is given by

$$\vec{F} = \sin y\hat{i} + x(1 + \cos y)\hat{j}$$

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the circular path given by $x^2 + y^2 = a^2$. 5

- (b) (i) Find the scalar potential of

$$\vec{A} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k} \quad 5$$

- (ii) If S be a closed surface and \vec{r} be the position vector of a point (x, y, z) with respect to an origin O , then prove that

$$\iint_S \frac{\vec{n} \cdot \vec{r}}{r^3} dS = 4\pi$$

when O lies inside of S . 5

A9/761

(Turn Over)

(4)

- (c) (i) Express $\vec{\nabla}\phi$ and $\vec{\nabla}\cdot\vec{A}$ in cylindrical coordinates. 2+3=5
- (ii) Prove that a spherical coordinate system is orthogonal. 5

GROUP—B

(Properties of Matter)

(Marks : 25)

5. Answer the following questions : 1×4=4

- (a) For a given material, the value of Young's modulus (Y) is 2.4 times of its shear modulus. Find the value of Poisson's ratio.
- (b) Define beam and cantilever.
- (c) What will be the surface tension of a liquid at critical temperature?
- (d) How does the velocity of propagation of waves passing over the surface of a liquid depend on gravity?

6. Answer the following questions : 2×3=6

- (a) If you ride your bicycle directly behind a large truck, you will find that you don't have to pedal hard to keep moving forward. Why?

(5)

- (b) Show that for an elastic material, the value of Poisson's ratio varies from -1 to 0.5.
- (c) Draw the resultant force of surface tension of a plane, concave and convex surface.

7. Answer any one of the following questions :

- (a) (i) Show that shear (θ) is equal to half a tensile and half a compressive strain at right angles to each other. 3
- (ii) Find the work done in stretching a wire of 1 sq. mm cross-section, Young's modulus $2 \times 10^{11} \text{ N/m}^2$ and 2 m long through 0.1 mm. 2
- (b) (i) Why is correction of Poiseuille's equation necessary? 3
- (ii) A rigid rod 1.5 m long is fixed horizontally at one end and loaded at the other by a mass of 0.1 kg. Calculate the depression of a point distant 1.2 m from the fixed end. Diameter of the rod is 2 cm. Young's modulus for the material of the rod is $1.01 \times 10^{11} \text{ N/m}^2$. 2

(6)

8. Answer either (a) or (b) :

- (a) (i) Establish the relation $Y = 3K(1 - 2\sigma)$, where Y is Young's modulus, K is bulk modulus and σ is Poisson's ratio.

The stress to strain ratio remains constant for a small deformation. What happens to this ratio, if deformation is made very large and why? 5+2=7

- (ii) Determine the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm. For glass, bulk modulus = $37 \times 10^9 \text{ Nm}^{-2}$. 3

- (b) (i) Using the relation for excess pressure acting on a curved surface

$$P = 2T \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

evaluate excess pressure for spherical liquid drop, air bubbles in liquid drop, soap bubble and cylindrical film.

Explain why the free surface of a liquid behaves like a stretched membrane. 4+3=7

(7)

- (ii) A liquid drop of radius R breaks into 64 tiny drops. Find the resulting change in energy in terms of surface tension T . 3
