

2019

STATISTICS

( Major )

Paper : 2.2

( Mathematical Method—I )

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. Answer the following as directed : 1×7=7

(a) Define gamma integral.

(b) In a finite interval which encloses no point of infinite discontinuity, the integrand is \_\_\_\_\_ and \_\_\_\_\_.

( Fill in the blanks )

(c) When does an integral

$$\int_0^1 x^{m-1}(1-x)^{n-1} dx$$

exist?

- (d) Define upper and lower Riemann sums of a function corresponding to the partition  $P$ .
- (e) Every uniformly convergent sequence is pointwise convergent and the uniform limit function is same as the pointwise limit function.

( Write True or False )

- (f) State the necessary condition for  $f(x)$  to have an extreme value at the point  $C$ .
- (g) If  $f(xy) = 2x^2 - xy + 2y^2$ , then find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  at the point  $(1, 2)$ .

2. Answer the following :

$2 \times 4 = 8$

- (a) State the geometrical (physical) interpretation of Cauchy's mean value theorem.
- (b) If a function  $f$  is twice derivable on  $[a, a+h]$ , then show that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a+\theta h),$$

$$0 < \theta < 1$$

- (c) Examine the convergence of

$$\int_0^1 \frac{dx}{\sqrt{1-x}}$$

- (d) Show that

$$\int_a^b (x-a)^{l-1} (b-x)^{m-1} dx = (b-a)^{l+m-1} \beta(l, m)$$

3. Answer any three questions :

$5 \times 3 = 15$

- (a) If a function  $f$  defined on  $[a, b]$  is (i) continuous on  $[a, b]$ , (ii) derivable on  $]a, b[$  and (iii)  $f(a) = f(b)$ , then prove that there exists at least one real number  $\xi$  between  $a$  and  $b$ , ( $a < \xi < b$ ) such that  $f'(\xi) = 0$ .
- (b) Show that the sequence  $\{f_n\}$ , where  $f_n(x) = x^n$ , is uniformly convergent on  $[0, k]$  and only pointwise convergent on  $[0, 1]$ .
- (c) If a function  $f$  defined on  $[a, a+h]$  is (i) continuous on  $[a, a+h]$  and (ii) derivable on  $]a, a+h[$ , then prove that there exists at least one number  $\theta \in ]0, 1[$  such that

$$f(a+h) = f(a) + hf'(a+\theta h), \quad \theta \in ]0, 1[$$

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(d) (i) Show that

$$\int_a^b \frac{dx}{(x-a)^P}$$

converges, if  $P < 1$  and diverges, if  $P \geq 1$ .

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(ii) Find the value of

$$\int_0^\pi \cos^4 \theta d\theta$$

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(e) Show that  $x^5 - 5x^4 + 5x^3 - 1$  has a maximum at  $x = 1$  and minimum at  $x = 3$  and neither at  $x = 0$ .

4. Answer any three questions :  $10 \times 3 = 30$

(a) (i) State and prove Cauchy's criterion for uniform convergence.

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(ii) Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem for the function

$$f(x) = x(x-1)(x-2) \text{ on } \left[0, \frac{1}{2}\right]$$

4

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(b) (i) Show that

$$\frac{\Gamma(z)\Gamma(a+1)}{\Gamma(z+a)} =$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{a(a-1)(a-2)\dots(a-n)}{n!} \frac{1}{z+n}$$

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(ii) Show that

$$\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(m, n)$$

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(c) (i) If  $\alpha, \beta, \gamma$  are the roots of the equation in  $t$ , such that

$$\frac{u}{a+t} + \frac{v}{b+t} + \frac{w}{c+t} = 1$$

then prove that

$$\frac{\partial(u, v, w)}{\partial(\alpha, \beta, \gamma)} = \frac{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)}{(b-c)(c-a)(a-b)}$$

6

(ii) Show that  $x^2$  is integrable on any interval  $[0, k]$ .

4

- (d) (i) Prove that the function  $f(xy) = |xy|^{\frac{1}{2}}$  is not differentiable at the point  $(0, 0)$ , but that  $f_x$  and  $f_y$  both exist at the origin and have the value zero. 5
- (ii) A function  $f$  is bounded and integrable on  $[a, b]$  and there exists a function  $\phi(x)$  such that  $\phi'(x) = f(x)$  on  $[a, b]$ , then prove that
- $$\int_a^b f(x) dx = \phi(b) - \phi(a) \quad 5$$
- (e) (i) Write a note on Lagrange's method of undetermined multiplier. 6
- (ii) If  $z = e^{xy^2}$ ,  $x = t \cos t$ ,  $y = t \sin t$ , find  $\frac{\partial z}{\partial t}$  at  $t = \frac{\pi}{2}$ . 4
- (f) State and prove Tylor's theorem for two variables. 10

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