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3 (Sem-1/CBCS) STA HC 2

2022

STATISTICS

(Honours)

Paper : STA-HC-1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : **(any ten)**

1×10=10

(a) If a function is derivable at all points of an interval except the ends points, it is said to be derivable in the open interval. (State True or False)

(b) The value of the integral

$$\int_0^1 x^{m-1} (1-x)^{n-1} dx$$

(i) $\beta(m, n)$

Contd.



(ii) $\beta(n, m)$

(iii) Both (i) and (ii)

(iv) None of the above

(Choose the incorrect option)

(c) The n th derivative of a^x is

(i) a^x

(ii) $(\log_e a)^n a^x$

(iii) na^x

(iv) None of the above

(Choose the correct option)

(d) Evaluate $\int_0^{\infty} e^{-3x} x^{\frac{1}{2}} dx$

(e) The differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 - 2\left(\frac{dy}{dx}\right)^2 + 5y = 0$$
 is of order _____ and degree _____.

(Fill in the blanks)

(f) State two properties of definite integrals.

(g) Define homogeneous function of two variables.

(h) If $f(x, y) = x^4 + xy + y^4$ find f_x and f_{yx} .

(i) The value of

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$$
 is

(i) ∞

(ii) 0

(iii) $\frac{\infty}{\infty}$

(iv) None of the above

(Choose the correct option)

(j) Write two properties of double integrals.

(k) The differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$$
 is called

(i) ordinary differential equation

(ii) partial differential equation

(iii) None of the above

(Choose the correct option)

(l) Find the differential equation of all the straight lines passing through the origin.

(m) Define beta integral of second kind.

(n) Lagrange's undetermined multipliers is a method of finding the _____ or _____ of a function subject to one or more conditions. (Fill in the blanks)

(o) Define Jacobian of the functions $u_1, u_2 \dots u_n$ with respect to $x_1, x_2 \dots x_n$.

(p) The value of $\Gamma(n+1)$ is

(i) $n!$

(ii) $n!^{-1}$

(iii) Both (i) and (ii)

(iv) $(n-1)!$

(Choose the incorrect option)

(q) The function x^n is continuous for all values of x when n is positive and continuous for all values of x except 0 when n is negative.

(State True or False)

(r) Define bounded function.

2. Answer **any five** of the following questions :
2×5=10

(a) Test the differentiability of the function

$$f(x) = \begin{cases} 1+x, & \text{if } x \leq 2 \\ 5-x, & \text{if } x > 2 \end{cases}$$

at $x=2$

(b) Find the n th differential coefficients of $\sin^3 x$.

(c) If $f(x) = x \cdot \frac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^x + e^{-x}}$, $x \neq 0$ and $f(0) = 0$,

show that $f(x)$ is continuous at $x=0$.

(d) Find the value of

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

(e) Show that $f(x) = 2x^3 - 21x^2 + 36x - 20$ has a maximum at $x=1$.

(f) Prove that

$$\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$$

(g) Solve the

$$\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - \frac{dy}{dx} + y = 0$$

(h) Prove that

$$\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta = \frac{\pi}{\sqrt{2}}$$

(i) If $x^3 + 3x^2y + 6xy^2 + y^3 = 1$, find $\frac{dy}{dx}$.

(j) If u be a homogeneous function of x and y of degree n , then show that

$$x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial x \partial y} = (n-1) \frac{\partial u}{\partial x}$$

3. Answer **any four** from the following questions : 5×4=20

(a) Show that the function

$f(x) = |x| + |x-1|$ is not differentiable at $x=1$ but differentiable at $n=2$.

(b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

(c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

(d) Find the solution

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$$

(e) If f_x and f_y are both differentiable at a point (a, b) of domain of definition of a function f , then

$$f_{xy}(a, b) = f_{yx}(a, b)$$

(f) Solve the differential equation

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^{5x}$$

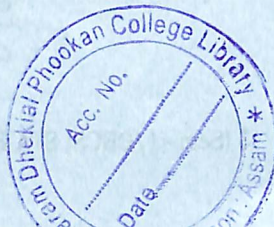
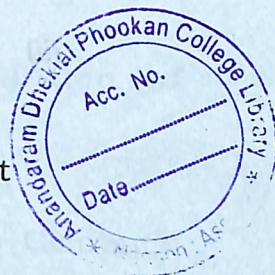
(g) If $0 < n < 1$, then show that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

Hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(h) Evaluate

$$\int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y) \, dy \, dx$$



4. Answer **any four** from the following questions : 10×4=40

(a) (i) Prove that

$$\int_{-a}^a f(x) dx = 0, \text{ if } f(x) \text{ is an odd}$$

function of x .

$$= 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ is an even function of } (x). \quad 3$$

(ii) Using properties of definite integral prove that

$$\int_0^{\pi/2} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2} \quad 7$$

(b) (i) Find the differential coefficient of $x^x + (\sin x)^{\log x}$ 5

(ii) If $y = e^x \log x$, show that in usual notation

$$xy_2 - (2x-1)y_1 + (x-1)y = 0 \quad 5$$

(c) (i) If $f(x,y) = \frac{2xy(x^2 - y^2)}{x^2 + y^2},$

$$(x,y) \neq (0,0)$$

$f(0,0) = 0$, find $f_x(0,0)$ and $f_y(0,0)$ 5

(ii) If $u = \log \left\{ \frac{x^2 + y^2}{x+y} \right\}$, prove that

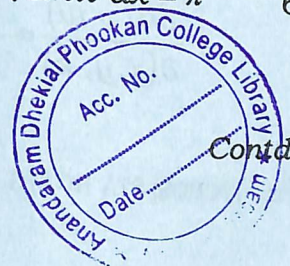
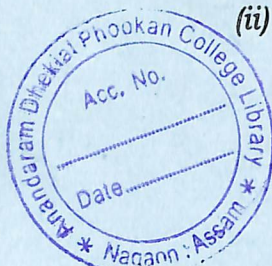
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1 \quad 5$$

(d) (i) Explain general solution of Clairaut's equation. 4

(ii) Solve the equation $(px - y)(x - yp) = 2p$ to Clairaut's form by the substitution $x^2 = u$, $y^2 = v$, and find its solution. 6

(e) (i) Show that

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} \, dx = \pi \quad 6$$



(ii) Prove that

$$\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}} \quad 4$$

(f) (i) Explain the procedure of equations solvable for p and y . 5

(ii) Solve the differential equation

$$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - 56 = 0 \quad 5$$

(g) (i) State the necessary and sufficient condition for extreme value of a function of two variables. 4

(ii) Find the maximum value of

$$f(x, y) = 3x^2 - y^2 + x^3 \quad 6$$

(h) Prove that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$$

(i) The roots of the equation

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ are u, v, w . Prove that

$$\frac{\partial(uvw)}{\partial(x, y, z)} = -2 \frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

(j) (i) Define partial differential equation. 1

(ii) Solve the partial differential equations

(A) $\left(\frac{y^2 z}{x}\right) p + xzq = y^2$ 5

(B) $xp + yq = z$ 4

