

Total number of printed pages-7

3 (Sem-4/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY-HC-4016

(Mathematical Physics-III)

Full Marks : 60

Time : Three hours

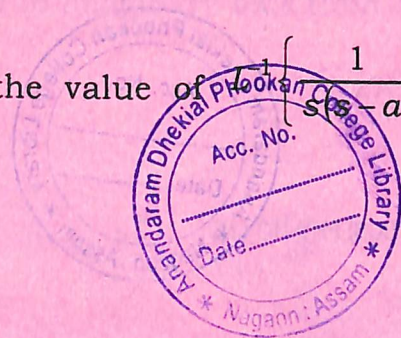
The figures in the margin indicate full marks for the questions.

1. Give short answers to the following questions : 1×7=7

(a) Find the principal value of i^i .

(b) Define a multiply connected region in complex plane.

(c) Find the value of $\int_a^{\infty} \frac{1}{s-a} ds$ for $s > a$.



Contd.

(d) What does the equation $|z-i|=2$ represent ?

(e) State convolution theorem of Fourier transform.

(f) Write the transformation rule for a covariant tensor of rank two.

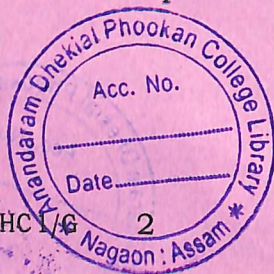
(g) Plot the number $e^{(1+i\frac{\pi}{4})}$.

2. Answer the following questions : $2 \times 4 = 8$

(a) Define simple pole and essential singularity.

(b) Establish the shifting property of Fourier transform.

(c) Find inverse Laplace transform of $t^{-\frac{1}{2}}$.



(d) Show that the number of independent components of a skew-symmetric tensor of rank 2 in n -dimensional space is

$$\frac{n(n-1)}{2}.$$

3. Answer **any three** questions of the following : $5 \times 3 = 15$

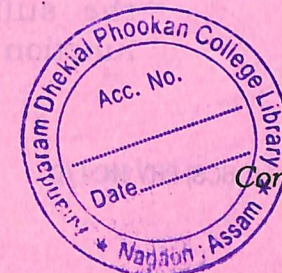
(a) Check the analyticity and hence find derivative of the function $f(z) = \sin z$.

$3 + 2 = 5$

(b) Find the value of the integral

$$\int_0^{1+i} (x-y-ix^2) dz \text{ along real axis from}$$

$z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$.



(c) Find the Fourier sine transform of a function defined by

$$f(t) = \begin{cases} \cos \alpha t, & 0 \leq t \leq \frac{\pi}{\alpha} \\ 0, & t > \frac{\pi}{\alpha} \end{cases}$$

(d) Evaluate :

$$L^{-1} \left\{ \frac{(s+1)}{s^2(s+2)^3} \right\}$$

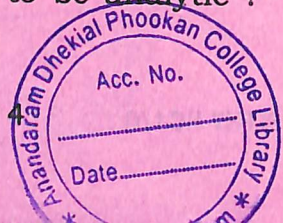
(e) Define Levi-Civita symbol in three dimensional space. Show that

$$(\bar{A} \times \bar{B})_i = \epsilon_{ijk} A_j B_k \quad 2+3=5$$

4. Answer the following questions : **(any three)**

10×3=30

(a) (i) Prove Cauchy-Riemann conditions for analytical functions. What is the sufficient condition for a function to be analytic ? 4+1=5



(ii) Show that $|z_1 + z_2| \leq |z_1| + |z_2|$

3

(iii) Give Laurent series expansion for function $f(z)$. 2

(b) (i) What are symmetric and antisymmetric tensors ? Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other antisymmetric in a pair of covariant or contravariant indices.

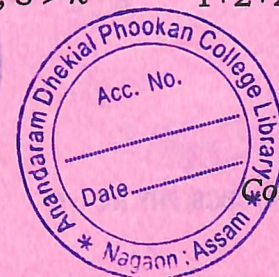
2+3=5

(ii) What is Kronecker delta ? Prove that Kronecker delta is a mixed tensor of second rank. 1+4=5

(c) (i) Define Laplace transform of a function $F(t)$. Show that

$$L(1) = \frac{1}{s}, \quad s > 0 \text{ and}$$

$$L(e^{kt}) = \frac{1}{s-k}, \quad s > k \quad 1+2+2=5$$



(ii) Find the inverse Laplace transform of

$$\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9} \quad 5$$

(d) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

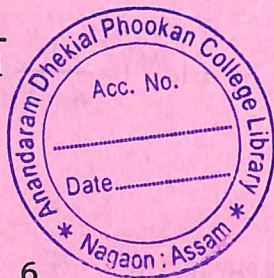
Hence, evaluate :

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$$

(e) Evaluate any two of the following integrals : $5 \times 2 = 10$

(i) $\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$

(ii) $\int_0^{\infty} \frac{dx}{x^2+1}$



(iii) $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$

(f) (i) The Laplace transform of $\sin 3t = \frac{3}{s^2+9}$ and the Laplace

transform of $\cos 5t = \frac{s}{s^2+25}$.

Find the Laplace transform of $5 \sin 3t + 3 \cos 5t$ using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of $4s + 5/(s-1)^2(s-2)$. 5

